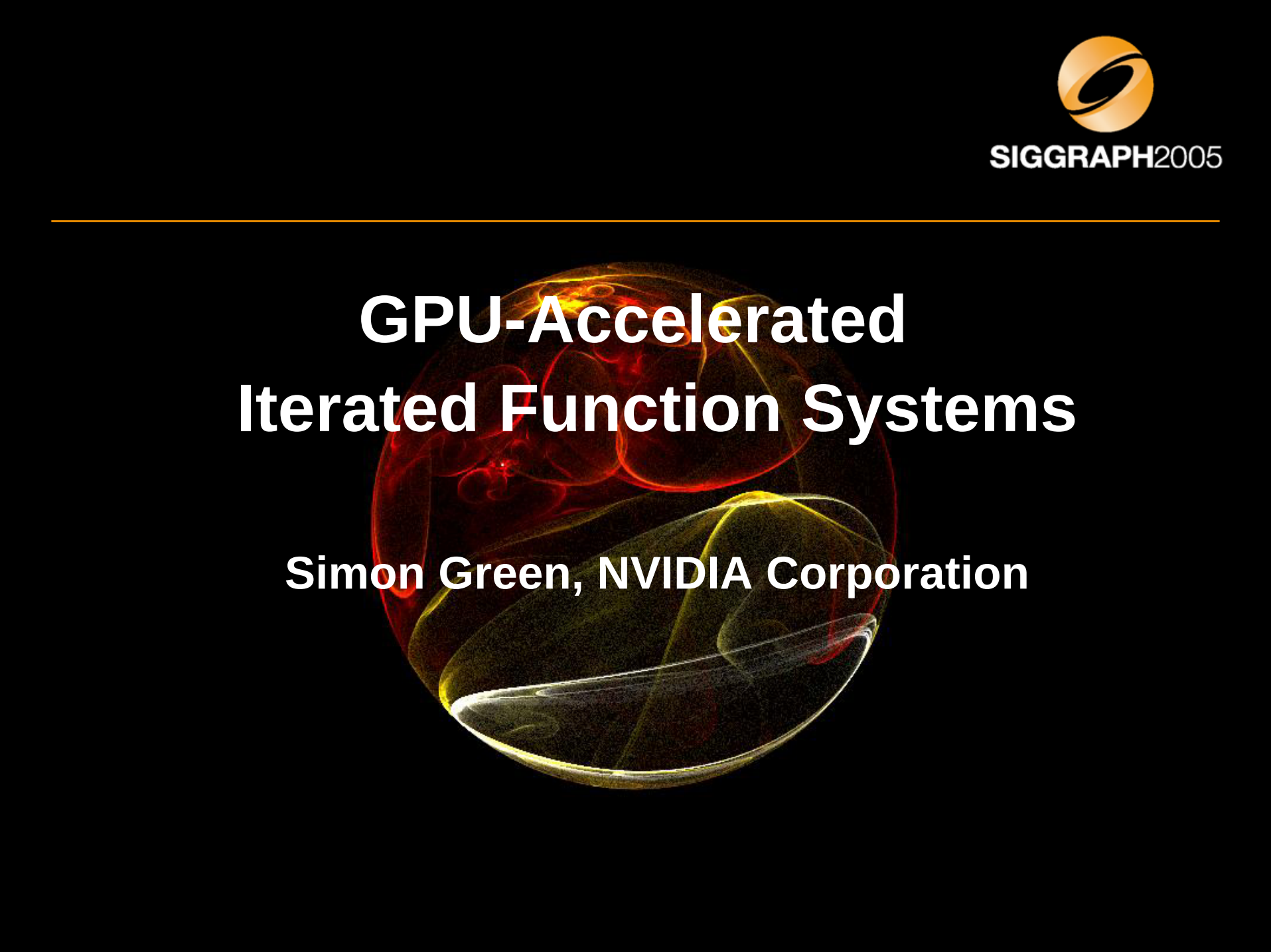


GPU-Accelerated Iterated Function Systems

Simon Green, NVIDIA Corporation

Iterated Function Systems

- Fractal
- Conceived by John Hutchinson (1981)
  - Popularized by Michael Barnsley (*Fractals Everywhere, 1998*)
- Consists of a set of functions
  - Functions map points from one point in space to another
  - Traditionally functions are affine transformations
  - Should be contractive (move points closer together)

IFS Mathematics

SIGGRAPH2005

- Image is set of points that are the solution to recursive set equation:

$$S = \bigcup_{i=0}^{n-1} f_i(S) \quad \text{where} \quad S \in R^2 \qquad f_i : R^2 \to R^2$$

$$f_i(x,y) = (a_i x + b_i y + c_i,\ d_i x + e_i y + f_i)$$

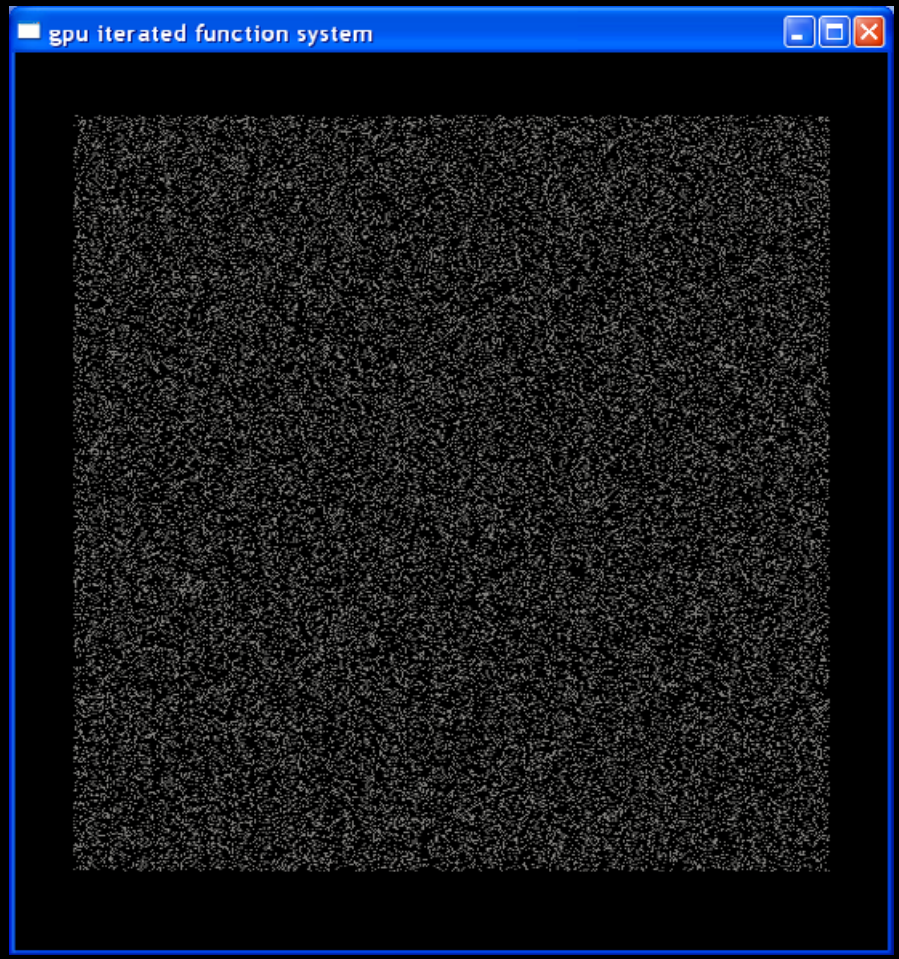
Generating Images of an IFS

- Commonly solved using the “chaos game”

```
Pick random point in space
Repeat {
  Pick function at random
  Apply function to point
  Plot the point
}
```

- **Converges to correct solution in ~20 steps**

Example: Sierpinski Triangle

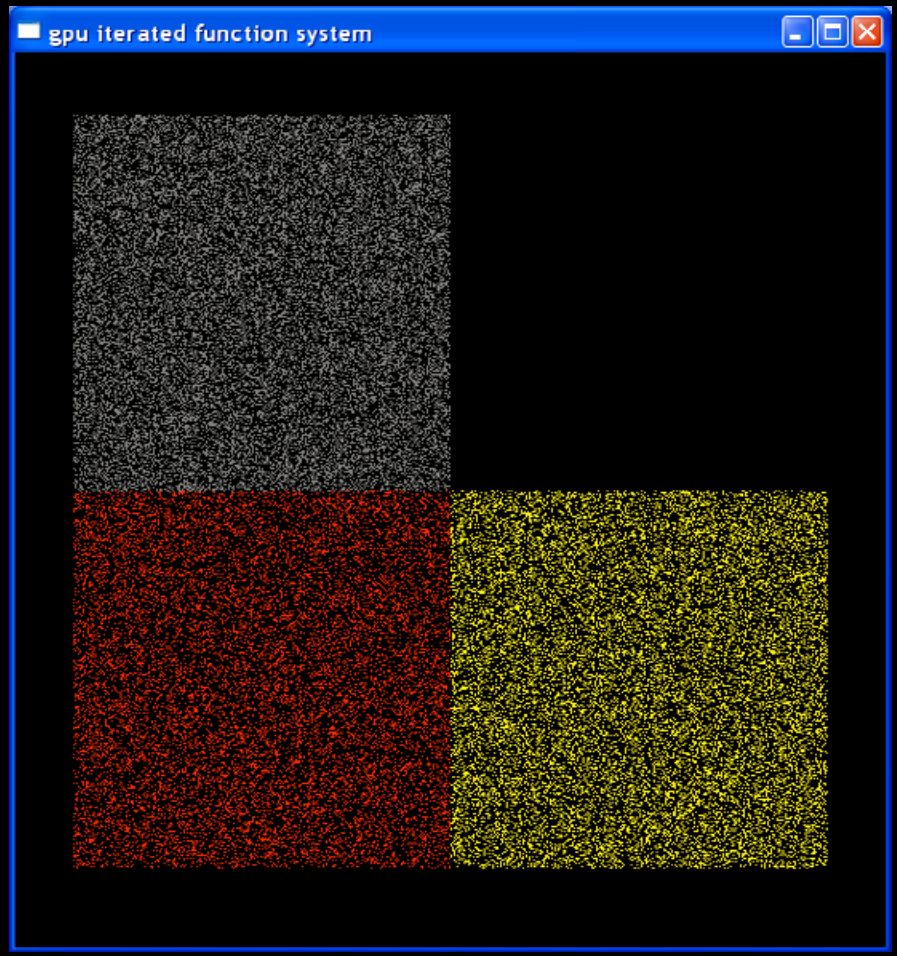


$$F_0(x, y) = (\frac{x}{2}, \frac{y}{2})$$

$$F_1(x, y) = (\frac{x+1}{2}, \frac{y}{2})$$

$$F_2(x, y) = (\frac{x}{2}, \frac{y+1}{2})$$

Example: Sierpinski Triangle

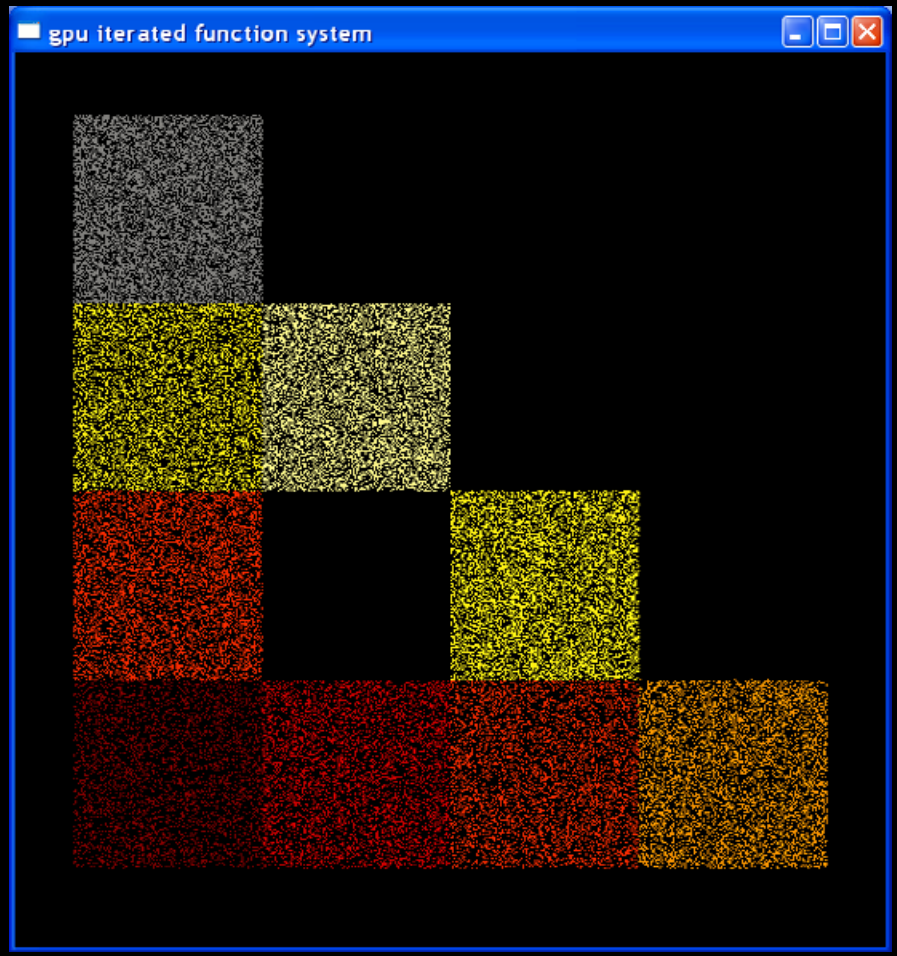


$$F_0(x, y) = (\frac{x}{2}, \frac{y}{2})$$

$$F_1(x, y) = (\frac{x+1}{2}, \frac{y}{2})$$

$$F_2(x, y) = (\frac{x}{2}, \frac{y+1}{2})$$

Example: Sierpinski Triangle

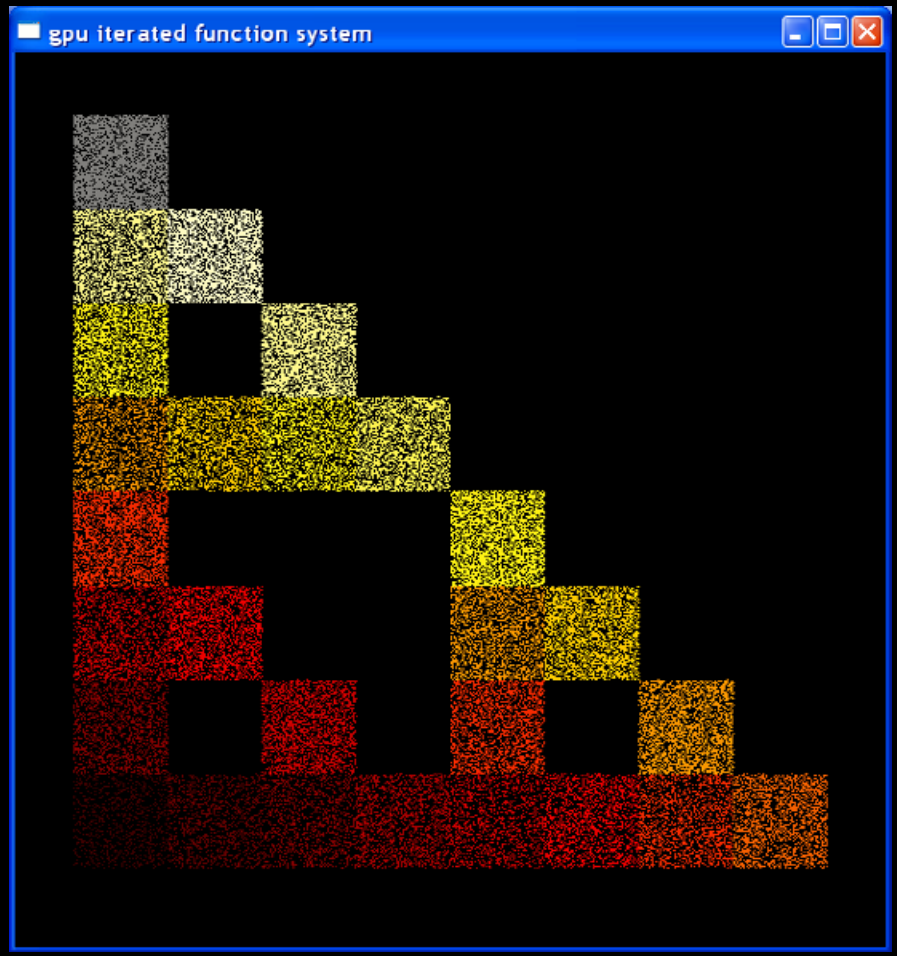


$$F_0(x, y) = (\frac{x}{2}, \frac{y}{2})$$

$$F_1(x, y) = (\frac{x+1}{2}, \frac{y}{2})$$

$$F_2(x, y) = (\frac{x}{2}, \frac{y+1}{2})$$

Example: Sierpinski Triangle

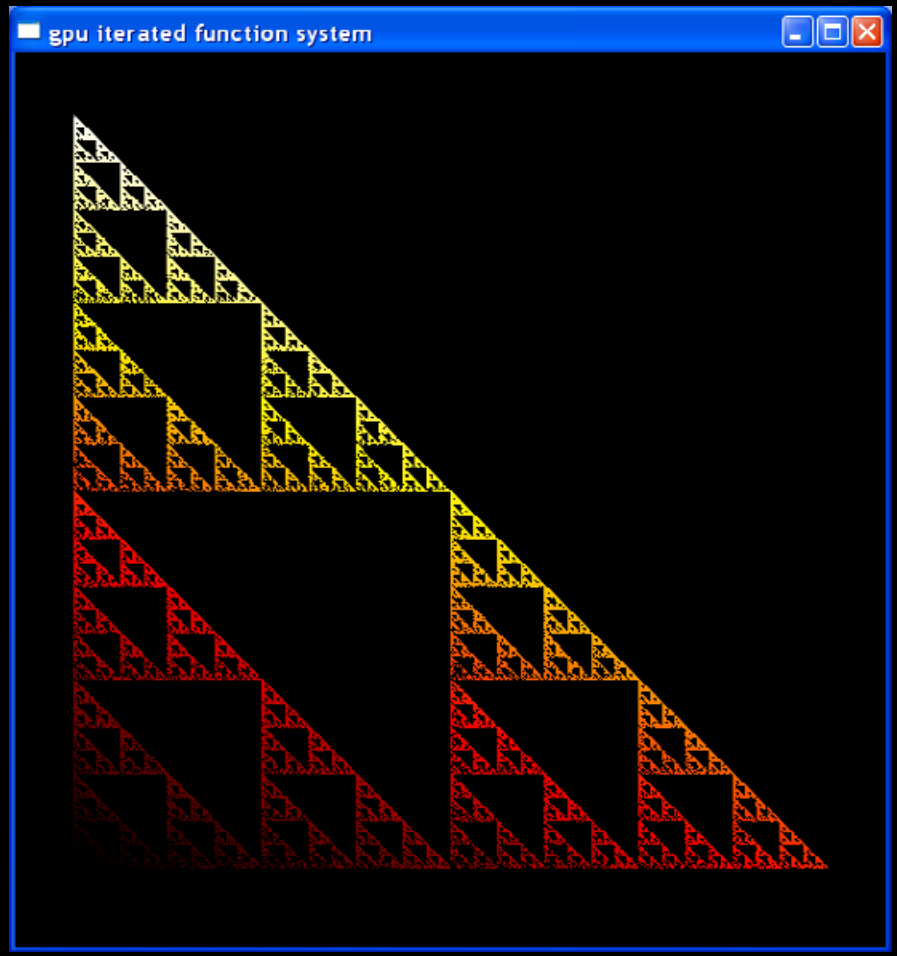


$$F_0(x,y) = (\tfrac{x}{2}, \tfrac{y}{2})$$

$$F_1(x,y) = (\tfrac{x+1}{2}, \tfrac{y}{2})$$

$$F_2(x,y) = (\tfrac{x}{2}, \tfrac{y+1}{2})$$

Example: Sierpinski Triangle



$$F_0(x, y) = (\frac{x}{2}, \frac{y}{2})$$

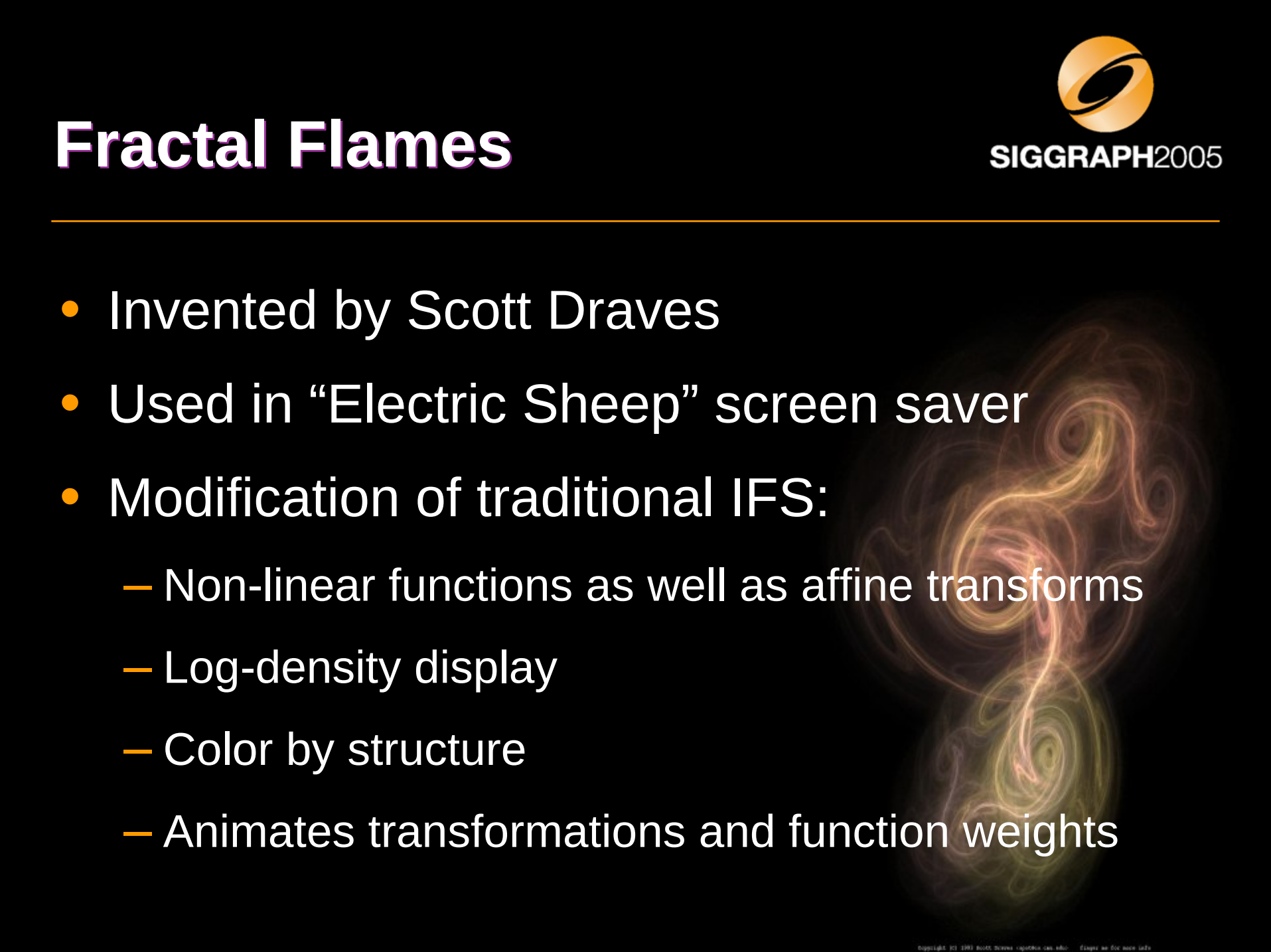
$$F_1(x, y) = (\frac{x+1}{2}, \frac{y}{2})$$

$$F_2(x, y) = (\frac{x}{2}, \frac{y+1}{2})$$

Fractal Flames

SIGGRAPH2005

- Invented by Scott Draves
- Used in “Electric Sheep” screen saver
- Modification of traditional IFS:
  - Non-linear functions as well as affine transforms
  - Log-density display
  - Color by structure
  - Animates transformations and function weights

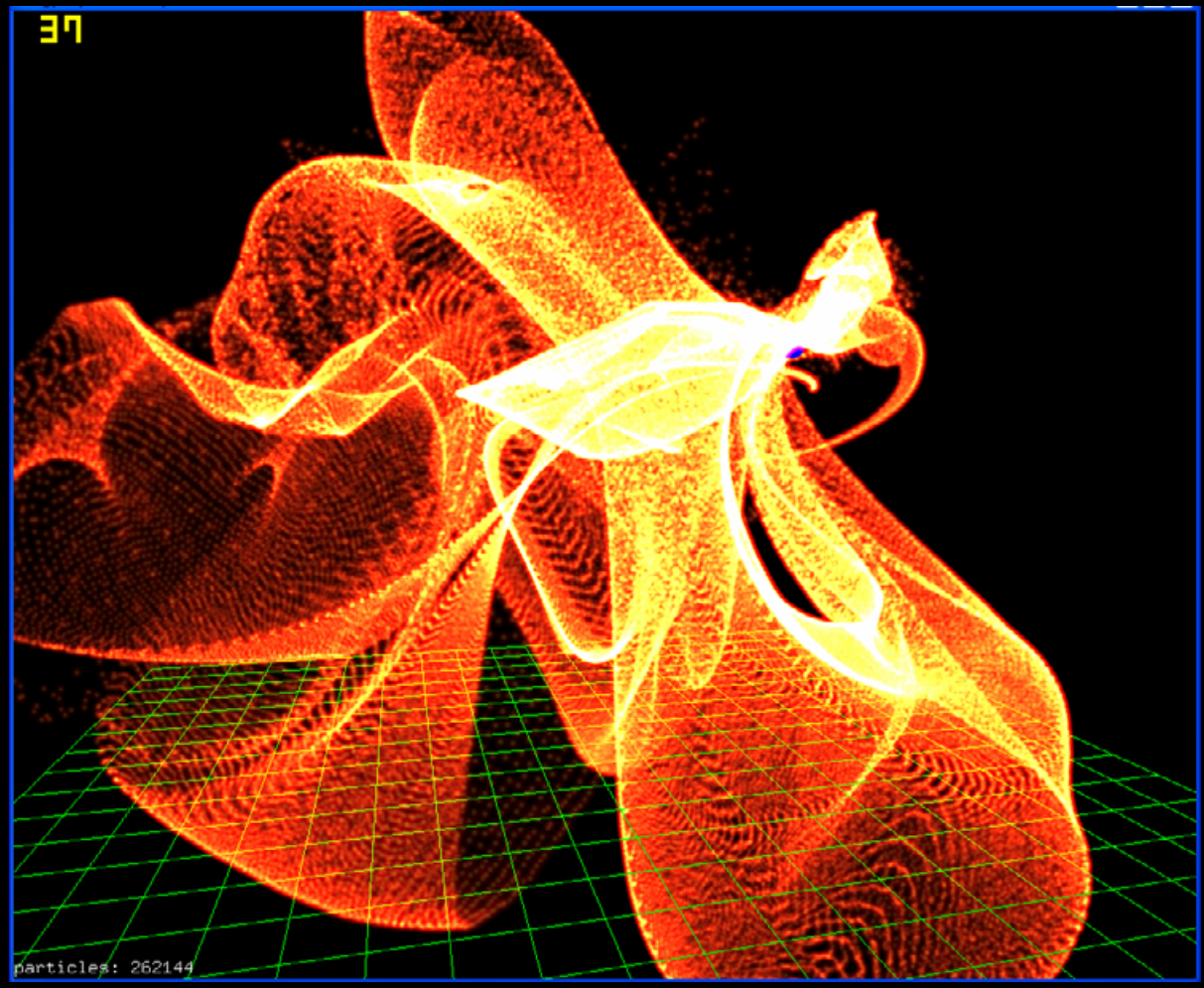


Fractal Flames

GPU Particle Systems

- Render-to-vertex array made particle systems possible on the GPU
  - Lutz Latta, “Building a Million Particle System”, GDC 2004
  - Kipfer et al, “Uberflow”, Graphics Hardware 2004
- Positions / velocities of particles stored in textures
- Simulation of particles computed using fragment programs
- Results rendered directly as point-sprites
  - No read-back to CPU required
- Enables millions of particles at interactive speeds

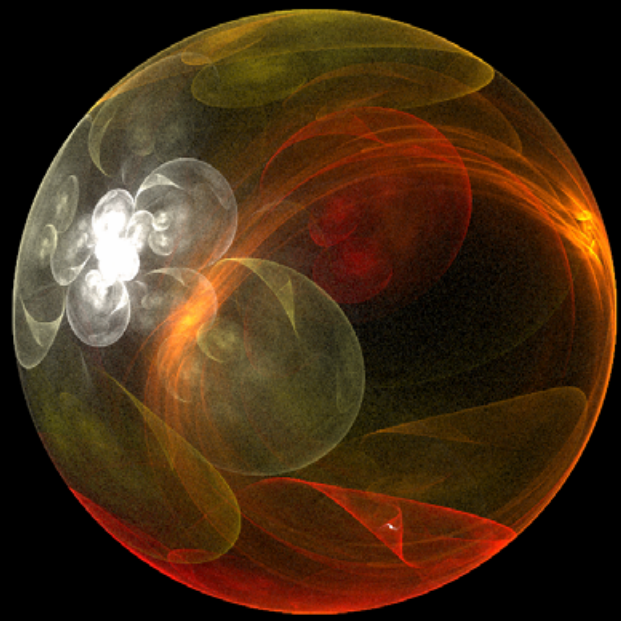
GPU Particle System Demo



GPUflame

SIGGRAPH2005

- Uses the GPU to accelerate computation and display of IFS fractals
- Extends functions to 3 dimensions
- Uses floating point blending for HDR display



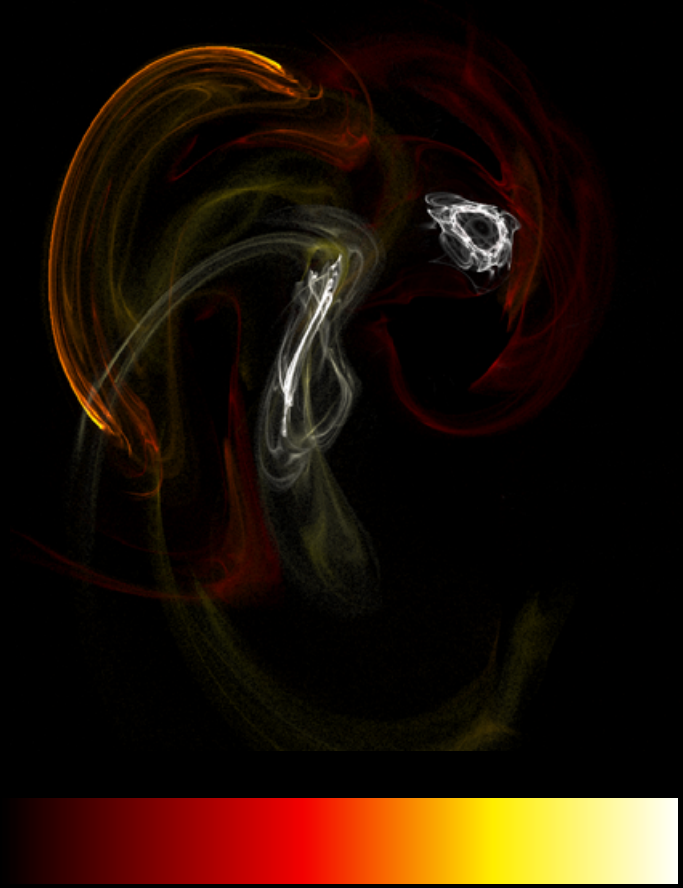
GPUflame Computation

- Uses standard GPGPU techniques
- Point positions stored in floating point texture
- Single quad drawn per iteration
- Fragment program applies IFS functions
  - Uses Cg language to describe functions
  - Random numbers come from pre-calculated texture

GPUflame Colouring

- Uses color index value
  - Tracks number of function applied at each step
  - Stored in alpha channel of position texture
- Value is used in final point shader to index into 1D palette texture

Colour Palettes



Colour Palettes

Colour Palettes

IFS Cg Code


```
// interface for variations
interface Variation {
    float3 func(float3 p, float r, float theta);
};

float3 linear(float3 p, float r, float theta)
{
    return p;
}

float3 spherical(float3 p, float r, float theta)
{
     return normalize(p);
}

float3 swirl(float3 p, float r, float theta)
{
    return float3(r*cos(theta+r), r*sin(theta+r), p.z);
}

…
```

IFS Cg Code


```
#include "variations.cg"

float4 main(in float2 uv : TEXCOORD0,
            uniform samplerRECT pos_tex,
            uniform samplerRECT rand_tex,
            uniform float time,
            uniform float4x4 matrix[],
            uniform Variation variation
            ) : COLOR
{

    // get position and color
    float4 tex = texRECT(pos_tex, uv);
    float3 p = tex.xyz;
    float c = tex.w;

    // get random number
    float4 rand = tex2D(rand_tex, (uv + float2(time, 0)));
```

IFS Cg Code


```
    // apply transformation
    int ri = floor(rand.x * matrix.length);
    for(int i=0; i<matrix.length; i++) {
        if (ri==i) {
            p = mul(matrix[i], float4(p, 1.0));
            ci = i / (float) (matrix.length-1);
        }
    }

    // apply function
    float r = length(p);
    float theta = atan2(p.x, p.y);
    p = variation.func(p, r, theta);

    c = (c + ci) / 2.0;     // mix old and new color
    return float4(p, c);
}
```

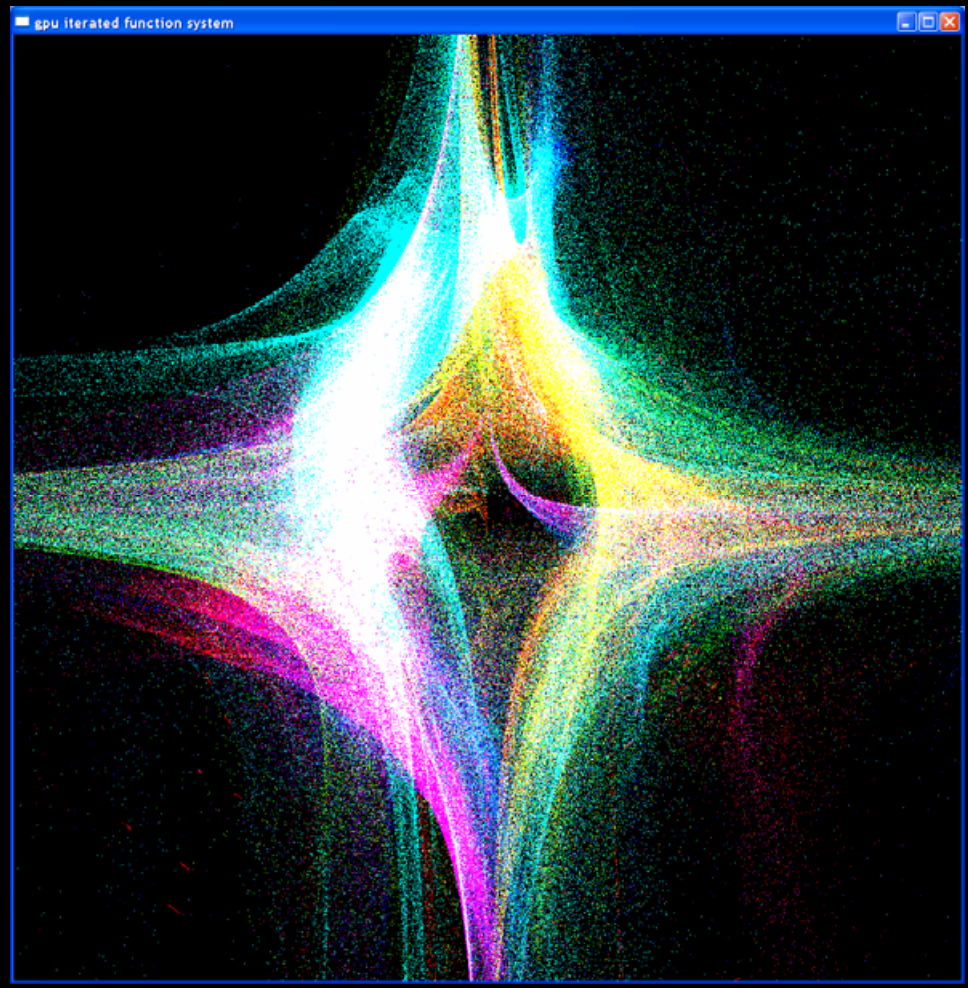
GPUflame Rendering

- Points rendered using render-to-vertex array (VBO/PBO OpenGL extensions)
  - Use of point sprites possible
- Rendered to 16-bit floating point buffer
  - Using additive blending

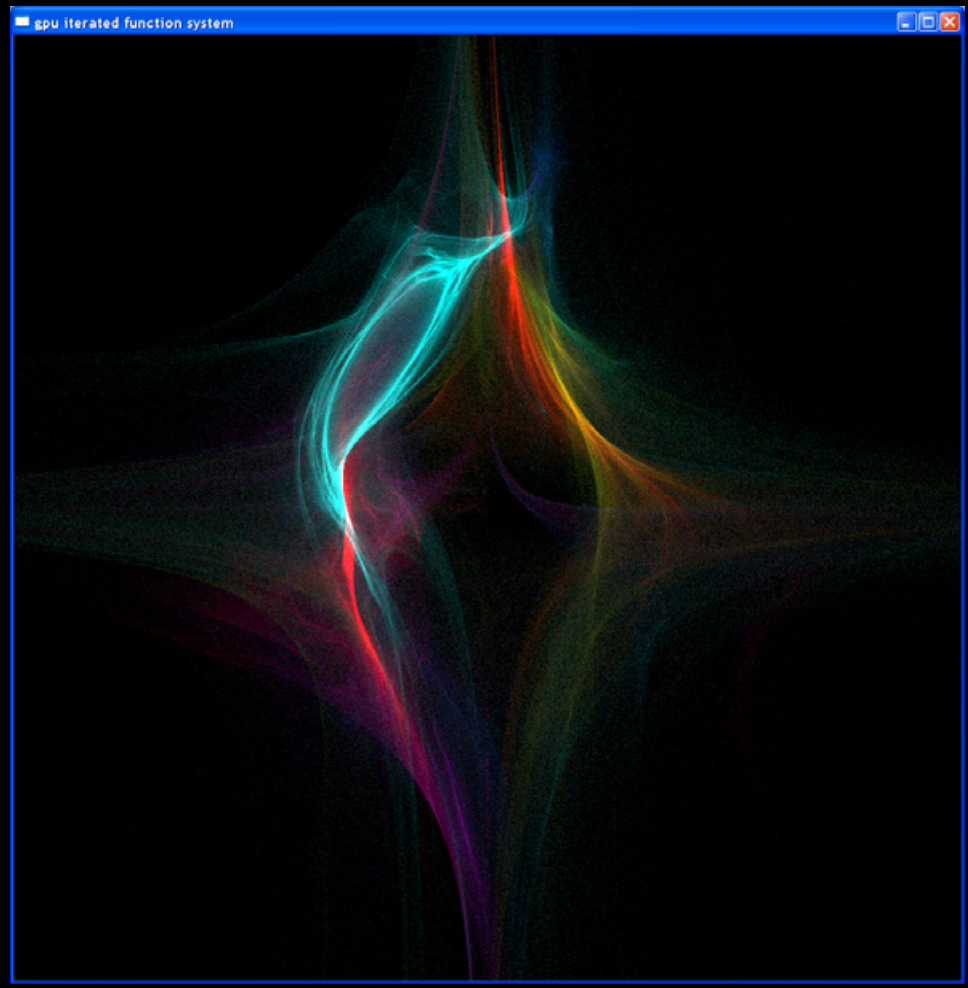
Tone Mapping

- Simple tone mapping pass
  - Converts floating point values to displayable 8-bit values
  - Exposure and gamma controls
- Glow effect
  - Perform Gaussian blur on image
  - Mix original and blurred image
  - Helps communicate very bright regions

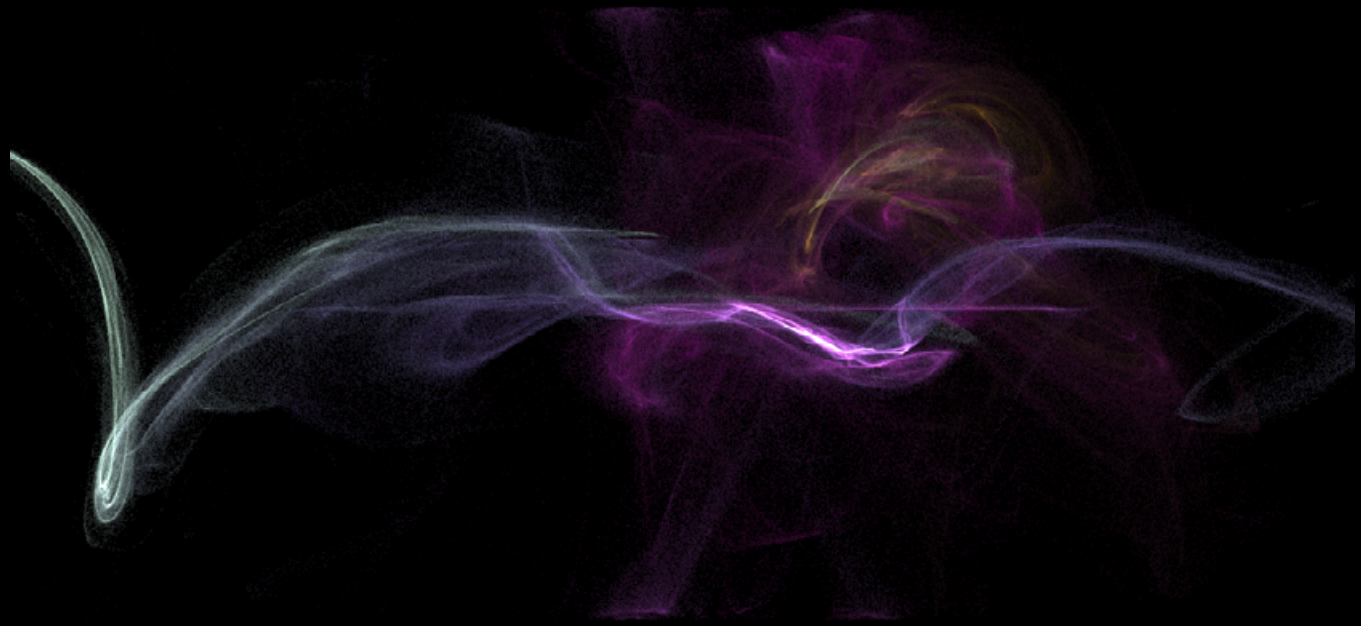
Without Tone Mapping



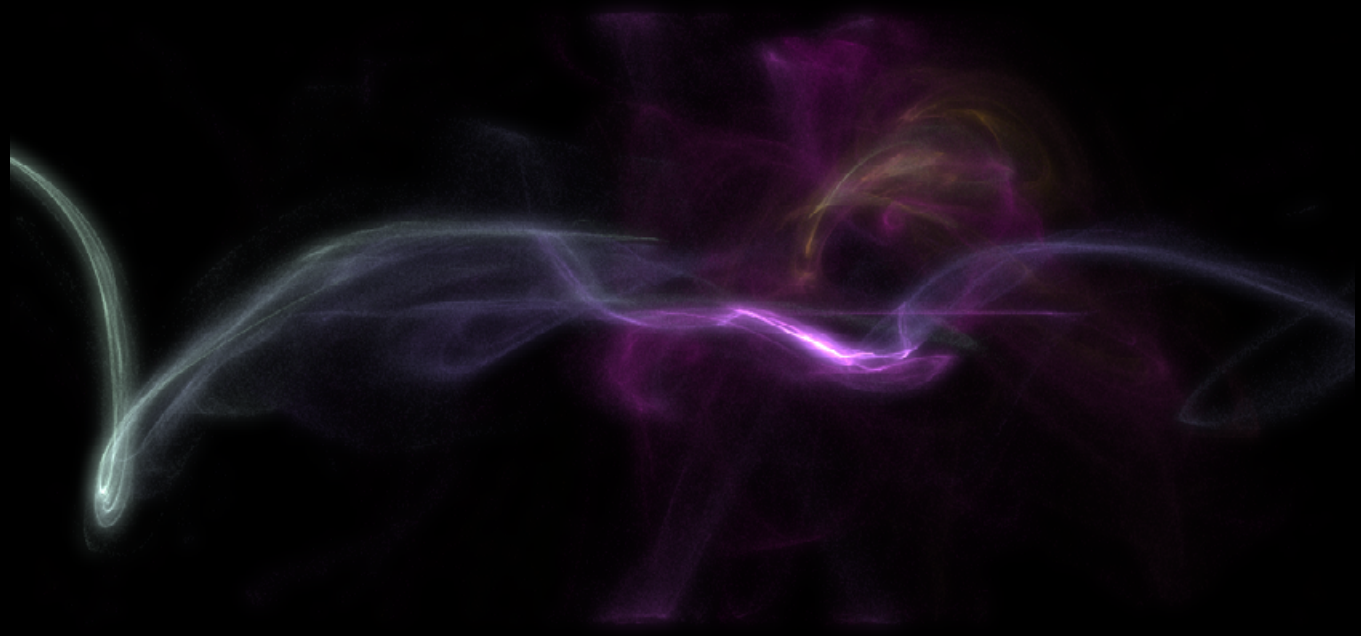
With Tone Mapping



Without Glow



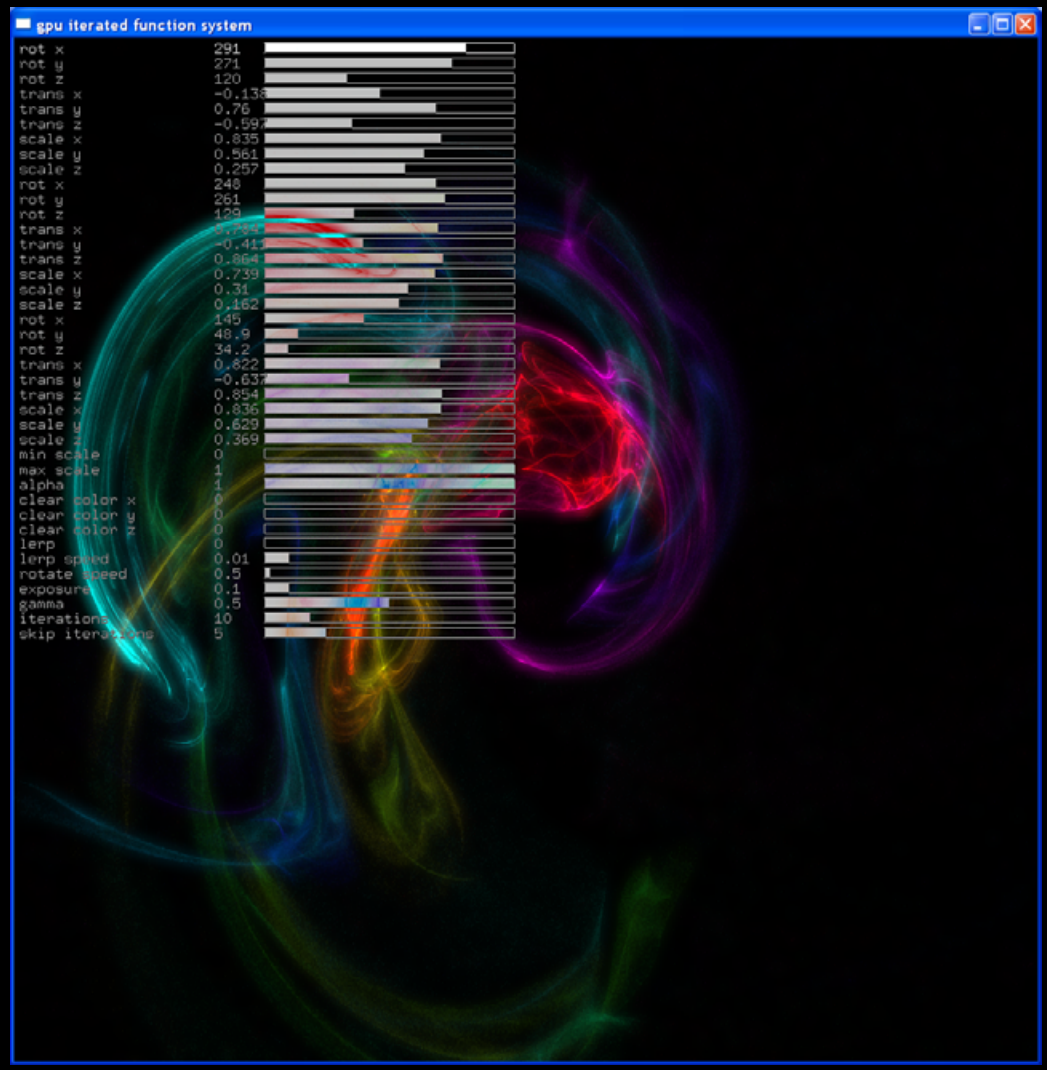
With Glow



Performance

- 1 million points @ 45 frames/sec
  - GeForce 6800 Go
  - 3 transforms, 1 function
- High quality rendering @ 10 frames/sec
  - 256k points, 10 passes
- Every function must be evaluated for each point

GPUflame Demo



Future Work

SIGGRAPH2005

- Analytical point anti-aliasing
- Tiled rendering for high resolution prints
- More complex colouring schemes
- New tone mapping algorithms
- New IFS functions

Conclusion

- GPU hardware allows interactive exploration of a complex mathematical space
- Questions?