



# Practical Metaballs and Implicit Surfaces

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## Agenda

- The idea and motivation
- ! Implementation details
- Caveats & optimizations
- Where to go from here
- Conclusion



#### What are isosurfaces?

- **Solution** Solution f(x, y, z) Defines a scalar field in 3D-space
- ∴ Isosurface S is a set of points for which f(x, y, z) = const
- f(x, y, z) = const can be thought of as an implicit function relating x, y and z
   Sometimes called implicit surfaces



#### What are isosurfaces?

- f(x, y, z) can come from Scattered data array Mathematical formula
- Isosurfaces are important data visualization tool

Medical imaging

Science visualization

Hydrodynamics

Cool effects for games!



#### Metaballs

- A particularly interesting case
- Use implicit equation of the form

$$\sum_{i=1}^{N} \frac{r_i^2}{\left\|\mathbf{x} - \mathbf{p}_i\right\|^2} = 1$$

Gradient can be computed directly

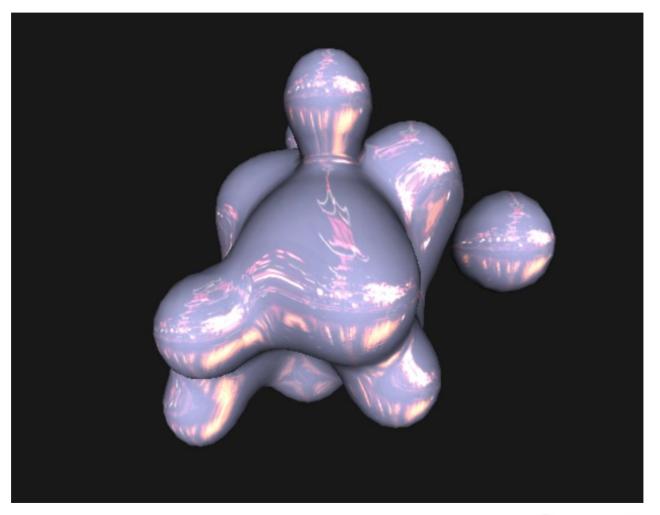
$$\mathbf{grad}(f) = -\sum_{i=1}^{N} \frac{2 \cdot r_i^2}{\|\mathbf{x} - \mathbf{p}_i\|^4} \cdot (\mathbf{x} - \mathbf{p}_i)$$

Soft/blobby objects that blend into each other Perfect for modelling fluids T1000-like effects





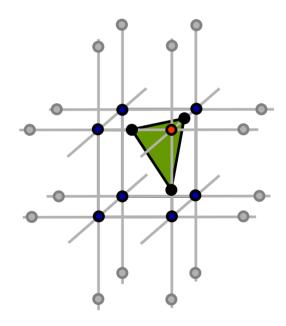
## Metaballs are cool!



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## The marching cubes algorithm

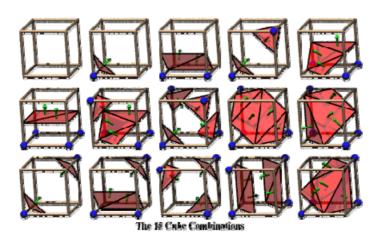
- A well-known method for scalar field polygonization
- Sample f(x, y, z) on a cubic lattice
- For each cubic cell...
  - Estimate where isosurface intersects cell edges by linear interpolation
  - Tessellate depending on values of f() at cell vertices



## The marching cubes algorithm

- Each vertex can be either "inside" or "outside"
- For each cube cell there are 256 ways for isosurface to intersect it

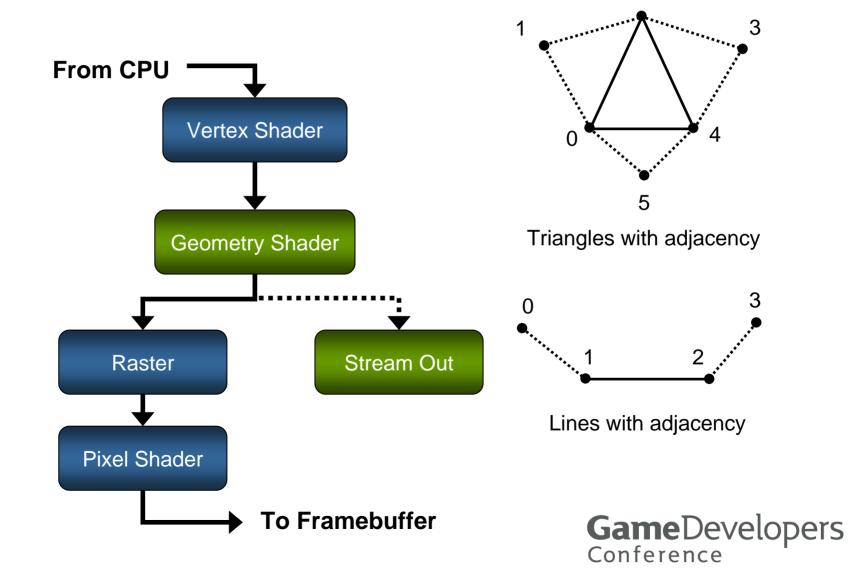
Can be simplified down to 15 unique cases





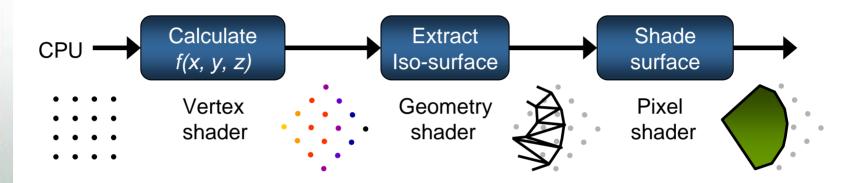


## Geometry shaders in DX10





## Implementation - basic idea



- App feeds a GPU with a grid of vertices
- VS transforms grid vertices and computes f(x, y, z), feeds to GS
- GS processes each cell in turn and emits triangles

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## A problem...

- Topology of GS input is restricted
  - Points
  - Lines
  - Triangles
  - with optional adjacency info
- Our "primitive" is a cubic cell

Need to input 8 vertices to a GS

A maximum we can input is 6 (with triangleadj)



#### Solution

First, note that actual input topology is irrelevant for GS

E.g. lineadj can be treated as quad input

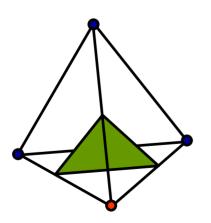
Work at tetrahedra level Tetrahedron is 4 vertices - perfect fit for lineadj!

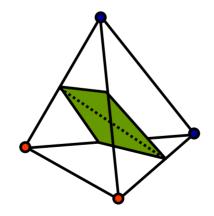
We'll subdivide each cell into tetrahedra

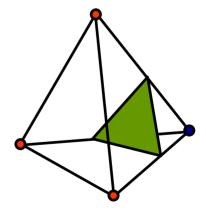


## Marching Tetrahedra (MT)

Tetrahedra are easier to handle in GS
 No ambiguities in isosurface reconstuction
 Always output either 1 or 2 triangles









## Generating a sampling grid

- There's a variety of ways to subdivide Along main diagonal into 6 tetrahedra – MT6 Tessellate into 5 tetrahedra – MT5 Body-centered tessellation – CCL
- Can also generate tetrahedral grid directly

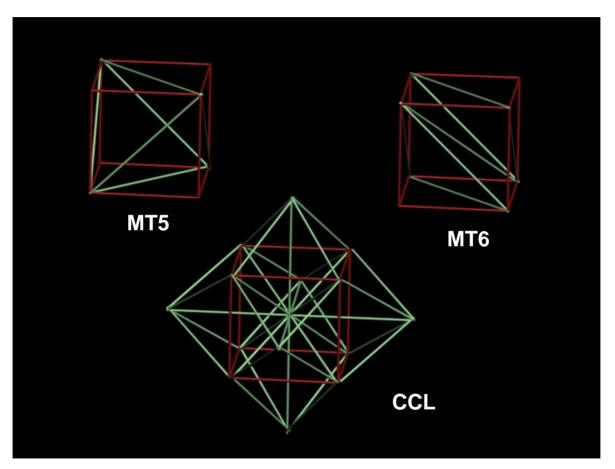
AKA simplex grid

Doesn't fit well within rectilinear volume





## Sampling grids





## Sampling grids comparison

	Generation Complexity	Sampling effectivene ss	Regularity
MT5	Med	Med	Low
MT6	Low	Med	Low
CCL	High	High	Med
Simplex	Low	Med	High



## VS/GS Input/output

#### Vertex Shader

```
// Metaball function
// Returns metaball function value in .w
// and its gradient in .xyz

float4 Metaball(float3 Pos, float3 Center, float RadiusSq)
{
   float4 o;
   float3 Dist = Pos - Center;
   float InvDistSq = 1 / dot(Dist, Dist);

   o.xyz = -2 * RadiusSq * InvDistSq * InvDistSq * Dist;
   o.w = RadiusSq * InvDistSq;

   return o;
}
```

#### Vertex Shader

```
#define MAX METABALLS
                          32
SampleData VS SampleField(float3 Pos : POSITION,
   uniform float4x4 WorldViewProj,
   uniform float3x3 WorldViewProjIT,
   uniform uint NumMetaballs, uniform float4 Metaballs[MAX METABALLS])
   SampleData o;
   float4 Field = 0;
   for (uint i = 0; i<NumMetaballs; i++)</pre>
        Field += Metaball(Pos, Metaballs[i].xyz, Metaballs[i].w);
   o.Pos = mul(float4(Pos, 1), WorldViewProj);
   o.N = mul(Field.xyz, WorldViewProjIT);
   o.Field = Field.w;
   o.IsInside = Field.w > 1 ? 1 : 0:
   return o;
```

## Geometry Shader

```
// Estimate where isosurface intersects grid edge
SurfaceVertex CalcIntersection(SampleData v0, SampleData v1)
{
    SurfaceVertex o;
    float t = (1.0 - v0.Field) / (v1.Field - v0.Field);
    o.Pos = lerp(v0.Pos, v1.Pos, t);
    o.N = lerp(v0.N, v1.N, t);
    return o;
}
```

## Geometry Shader

```
[MaxVertexCount(4)]
void GS TesselateTetrahedra(lineadj SampleData In[4],
                             inout TriangleStream<SurfaceVertex> Stream)
   // construct index for this tetrahedron
   uint index =
          (In[0].IsInside << 3) | (In[1].IsInside << 2) |
          (In[2].IsInside << 1) | In[3].IsInside;
   const struct { uint4 e0; uint4 e1; } EdgeTable[] = {
          { 0, 0, 0, 0, 0, 0, 0, 1 }, // all vertices out
          { 3, 0, 3, 1, 3, 2, 0, 0 }, // 0001
          { 2, 1, 2, 0, 2, 3, 0, 0 }, // 0010
          { 2, 0, 3, 0, 2, 1, 3, 1 }, // 0011 - 2 triangles
          { 1, 2, 1, 3, 1, 0, 0, 0 }, // 0100
          { 1, 0, 1, 2, 3, 0, 3, 2 }, // 0101 - 2 triangles
          { 1, 0, 2, 0, 1, 3, 2, 3 }, // 0110 - 2 triangles
          { 3, 0, 1, 0, 2, 0, 0, 0 }, // 0111
          { 0, 2, 0, 1, 0, 3, 0, 0 }, // 1000
          { 0, 1, 3, 1, 0, 2, 3, 2 }, // 1001 - 2 triangles
          { 0, 1, 0, 3, 2, 1, 2, 3 }, // 1010 - 2 triangles
          { 3, 1, 2, 1, 0, 1, 0, 0 }, // 1011
          { 0, 2, 1, 2, 0, 3, 1, 3 }, // 1100 - 2 triangles
          { 1, 2, 3, 2, 0, 2, 0, 0 }, // 1101
          { 0, 3, 2, 3, 1, 3, 0, 0 } // 1110
   };
```

## Edge table construction

```
const struct { uint4 e0; uint4 e1; } EdgeTable[] = {
        \{ 3, 0, 3, 1, 3, 2, 0, 0 \}, // index = 1
Index = 0001,
i.e. vertex 3 is "inside"
```

## Geometry Shader

```
// ... continued
// don't bother if all vertices out or all vertices in
if (index > 0 && index < 15)
     uint4 e0 = EdgeTable[index].e0;
     uint4 e1 = EdgeTable[index].e1;
     // Emit a triangle
     Stream.Append(CalcIntersection(In[e0.x], In[e0.y]));
     Stream.Append(CalcIntersection(In[e0.z], In[e0.w]));
     Stream.Append(CalcIntersection(In[e1.x], In[e1.y]));
     // Emit additional triangle, if necessary
     if (e1.z != 0)
     Stream.Append(CalcIntersection(In[e1.z], In[e1.w]));
```

## Respect your vertex cache!

- f(x, y, z) can be arbitrary complex
  E.g., many metaballs influencing a vertex
- Need to be careful about walk order Worst case is 4x more work than necessary! Straightforward linear work is not particularly cache friendly either
- Alternatively, can pre-transform with StreamOut



## Respect your vertex cache!

- Can use space-filling fractal curves Hilbert curve Swizzled walk
- We'll use swizzled walk
- To compute swizzled offset, just interleave x, y and z bits

$$\mathbf{x} = x_1 x_0$$

$$\mathbf{y} = y_3 y_2 y_1 y_0$$

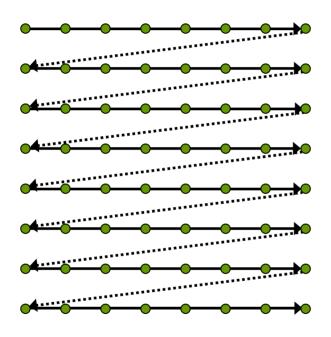
$$\mathbf{z} = z_2 z_1 z_0$$

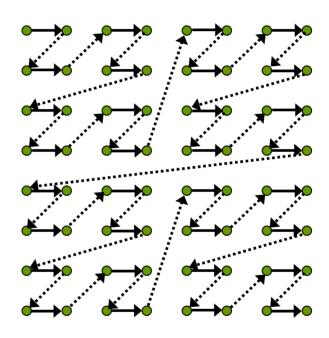
$$swizzle(\mathbf{x}, \mathbf{y}, \mathbf{z}) = y_3 z_2 y_2 z_1 y_1 x_1 z_0 y_0 x_0$$





#### Linear walk vs swizzled walk





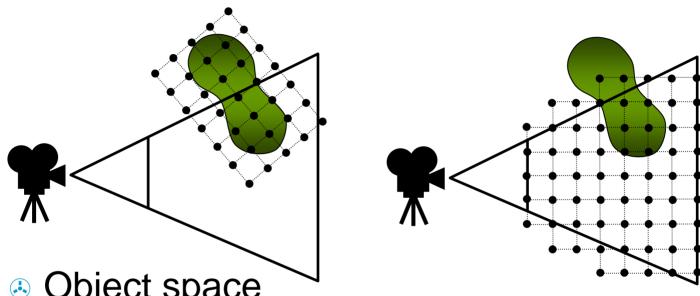
Linear walk

Swizzled walk





## Tessellation space



Object space

Works if you can calculate BB around your metaballs

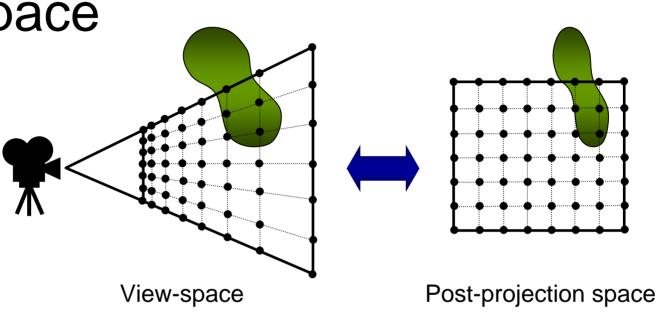
View space

Better, but sampling rate is distributed inadequately

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# Tessellation in post-projection space



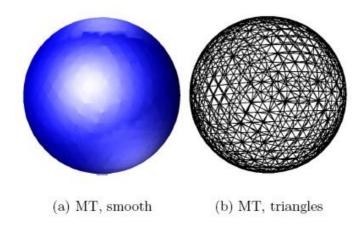
Post-projective space Probably the best option We also get LOD for free!



# Problems with current approach

- Generated mesh is over-tessellated General problem with MT algorithms
- Many triangles end up irregular and skinny

Good sampling grid helps a bit



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#### Possible enhancements

- Regularized Marching Tetrahedra (RMT) Vertex clustering prior to polygonization Generated triangles are more regular For details refer to [2]
- Need to run a pre-pass at vertex level, looking at immediate neighbors
  For CCL, each vertex has 14 neighbors
  GS input is too limited for this ②



## More speed optimizations

- Can cull metaballs based on ROI
  Only 3 or 4 need to be computed per-vertex
- Can use bounding sphere tree to cull Re-compute it dynamically on a GPU as metaballs move
- Cool effect idea particle system metaballs

Mass-spring can also be interesting





#### Conclusion

- DX10 Geometry Shader can be efficiently used for isosurface extraction
- Allows for class of totally new cool effects Organic forms with moving bulges GPGPU to animate metaballs Add noise to create turbulent fields Terminator2 anyone?





#### References

[1] J.Patera, V.Skala "Centered Cubic Lattice Method Comparison"

[2] G.M.Treece, R.W.Prager and A.H.Gee "Regularised Marching Tetrahedra: improved iso-surface extraction"





### Questions?

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